A Note on Regularization methods in Kaluza-Klein Theories

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Abstract

We comment on the presence of power-like divergences in Kaluza-Klein theories with supersymmetry breaking a là Scherk-Schwarz. By introducing a SUSY preserving regulator, we show that, in the context of a recently model proposed by Barbieri, Hall and Nomura, the 1-loop Higgs mass induced by Yukawa interactions is finite and unambiguously defined. The same result applies to similar models.

Recently there has been a growing interest in extensions of the Standard Model with extra compact dimensions of Tev size in which matter and gauge fields propagate in the bulk [1,2]. One key ingredient of these models is the breaking of supersymmetry by a mechanism a là Scherk-Schwarz [3] where boundary conditions are responsible for the mismatch between bosonic and fermionic sectors in the low energy spectrum. In this scenario the SUSY breaking is soft and the radiative corrections to scalar masses are expected to be free from power-like divergences.

In [2] Barbieri, Hall and Nomura constructed a realistic model based on a supersymmetric 5D extension of the SM in which the fifth dimension is compactified to $S^1/(Z_2 \times Z_2')$. One of the main features is that the Higgs mass turns out to be finite and negative at one loop, triggering radiatively EWSB. Notice that even if for any fixed energy the particle spectrum is different for bosons and fermions, the dynamics is still supersymmetric determining the cancellation of power-like divergences. Consequently, in order to perform a sensible computation, one has to sum over the entire KK tower [2,4].

In [5] doubts has been raised on this picture suggesting that the radiative corrections are finite because of a subtle fine-tuning hidden in the Kaluza-Klein regularization. The argument in [5] makes use of a sharp cutoff both in the KK sum and in the momentum integral and this causes a hard breaking of the supersymmetry.

In this letter we reproduce the result in [2] by using a Pauli-Villars (PV) regulator which manifestly preserves supersymmetry, showing that there is no UV ambiguity in $m^2_{\phi_H}$. We have also repeated the computation in dimensional regularization which is simpler to implement, obtaining the same result. We stress that our conclusions can be extended to similar models in which supersymmetry is broken a là Scherk-Schwarz.

We recall that in [2] all the Standard Model fields live in 5D, in particular the matter fields (H, Q, U, D, L, E) are described by a five dimensional hypermultiplet consisting of two 4D chiral superfields (Φ_i, Φ_i^c) with $\Phi_i = (H, Q, U, D, L, E)$. As a consequence of the orbifold projections, the $N = 2$ bulk SUSY reduces at the two fixed points $y = 0$ and $y = \pi R/2$ to two different $N = 1$ 4D supersymmetries, S and S' respectively. The relevant interaction for the one loop correction to the Higgs mass is the top Yukawa coupling, localized on the brane sitting in the orbifold fixed point $y = 0$ (identified with $y = \pi R$). The PV regulator is introduced at the Lagrangian level by adding an higher derivative term in the kinetic part [6]. A term like $\Box_5^2$, or $\Box^2$, where $\Box$ represents the 4D box and $\Box_5 = \Box - \partial_5^2$, is sufficient to regulate our integrals and preserves N=2 supersymmetry in the bulk.

The cancellation of divergences between the bosonic and fermionic contribution is most easily exhibited using $\Box^2$; we will show explicitly how the two different choices lead to the same conclusions. Following [7] we write the PV regularized action in terms of 4D chiral superfields as

$$S = S_{kin} + S_{int} \tag{1}$$

$$\begin{aligned} S_{kin} = \sum_i \int d^5x \Big\{ \int d^2\theta d^2\bar\theta \; \bar\Phi_i \left[1 + \Lambda^{-4}\Box^2\right] \Phi_i + \bar\Phi_i^c \left[1 + \Lambda^{-4}\Box^2\right] \Phi_i^c \\ + \int d^2\theta \; \Phi_i^c \, \partial_5 \left[1 + \Lambda^{-4}\Box^2\right] \Phi_i \Big\} \end{aligned} \tag{2}$$

$$S_{int} = \int d^5x \, \frac{1}{2} \left[\delta(y) + \delta(y - \pi R)\right] \int d^2\theta \left(\lambda_U \, QUH + \text{h.c.}\right) \tag{3}$$

From this expression it is manifest that the action (1) is invariant under the 4D $N = 1$ supersymmetry still operating at $y = 0$. In terms of the components fields, after a KK decomposition, the Yukawa interaction (3) reads

$$S_{int} = \frac{f_t}{\sqrt{2}} \sum_{k,l=0}^{\infty} \int d^4x \left(F_Q^{(k)} \phi_U^{(l)} \phi_H + F_U^{(k)} \phi_Q^{(l)} \phi_H - \eta_k \eta_l \, \psi_Q^{(k)} \psi_U^{(l)} \phi_H + \text{h.c.} \right) + \cdots \tag{4}$$

where $(\phi_i^{(n)}, \psi_i^{(n)}, F_i^{(n)})$ denote the n-th mode component fields of Φ_i, ϕ_H is the zero mode Higgs field, $f_t = (\lambda_U)_{33}/(\pi R)^{3/2}$ and $\eta_k = 1/\sqrt{2}$ for $k = 0$, $\eta_k = 1$ for $k \neq 0$. Notice that in the presence of the regulator the auxiliary fields F_i propagate and they cannot be eliminated. The relevant Feynman diagrams

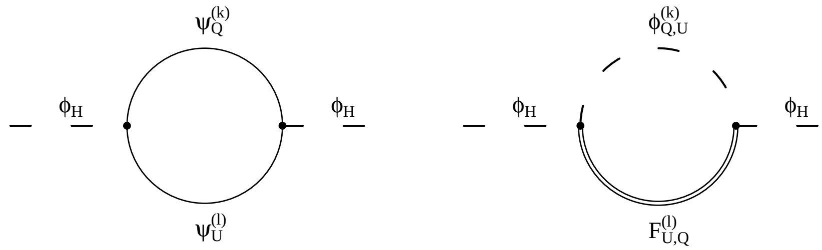

Fig. 1. *One loop contribution to the Higgs mass from the top Yukawa coupling.*

contributing to the Higgs mass $m^2_{\phi_H}$ are shown in figure 1. The result is

$$\begin{aligned} -im^2_{\phi_H} = \frac{iN_c f_t^2}{4R^2} \int \frac{d^4x}{(2\pi)^4}\, x^2\, \frac{(\Lambda R)^8}{\left[(\Lambda R)^4 + x^4\right]^2} \times \\ \left\{ \left[\sum_{k=-\infty}^{+\infty} \frac{1}{(x^2 + (2k)^2)} \right]^2 - \left[\sum_{k=-\infty}^{+\infty} \frac{1}{(x^2 + (2k+1)^2)} \right]^2 \right\}, \end{aligned} \tag{5}$$

We stress that our regulator makes the standing alone contribution from the bosonic (fermionic) sector in (5) convergent for any finite Λ as one can easily verify by simple power counting. Because of this, it is clear that exchanging the series with the integral is a legitimate operation, leading in both cases to a convergent result for the single contribution. Resumming the series first is simpler and gives

$$-im^2_{\phi_H} = \frac{iN_c f_t^2}{R^2} \frac{\pi^2}{16} \int \frac{d^4x}{(2\pi)^4} \frac{(\Lambda R)^8}{\left[(\Lambda R)^4 + x^4\right]^2} \left\{ \coth^2\left[\frac{\pi x}{2}\right] - \tanh^2\left[\frac{\pi x}{2}\right] \right\}, \tag{6}$$

The bosonic and fermionic parts both contain a term Λ^4 which exactly cancels in the difference, leaving a finite result. This can be explicitly seen writing

$$\begin{aligned} m^2_{\phi_H}\Big|_{fer} &= -\frac{N_c f_t^2}{128R^2} \left\{ \frac{(\Lambda R)^4}{4} + \int_0^\infty dx\, x^3 \left(\coth^2\left[\frac{\pi x}{2}\right] - 1 \right) + \mathcal{O}\left(\frac{1}{\Lambda}\right) \right\} \\ m^2_{\phi_H}\Big|_{bos} &= +\frac{N_c f_t^2}{128R^2} \left\{ \frac{(\Lambda R)^4}{4} + \int_0^\infty dx\, x^3 \left(\tanh^2\left[\frac{\pi x}{2}\right] - 1 \right) + \mathcal{O}\left(\frac{1}{\Lambda}\right) \right\} \end{aligned} \tag{7}$$

Notice that a term Λ^2 doesn't appear because it would correspond to a non-local contribution which cannot be canceled by a counterterm on the brane. Indeed, locality forces the counterterm to be localized on the boundary and, by simple power counting, it must be proportional to Λ^4, having the 5D Higgs field dimension $[\text{mass}]^{3/2}$ and the Yukawa λ_U dimension $[\text{mass}]^{-3/2}$. In other words, the effective field theory restricts the possible forms of the divergences, leaving only the Λ^4 term. Summing up the fermionic and bosonic contributions

in (7) we obtain

$$m^2_{\phi_H} = -\frac{21\zeta(3)}{64\pi^4}\,\frac{N_c f_t^2}{R^2} \tag{8}$$

which coincides with the result in [2].

Choosing from the beginning the 5D box, instead of the 4D one for the PV regulator in (2), leads to the same conclusion, but in this case expressions are more involved. Resumming the series first, one obtains

$$-im^2_{\phi_H} = -\frac{iN_c f_t^2}{8R^2}\int \frac{d^4x}{(2\pi)^4}\,x^2\left[2f^2(x,\Lambda R) - f(x,\Lambda R)f(x/2,\Lambda R/2)\right] \quad , \tag{9}$$

where

$$f(x,\Lambda R) = \pi\,\frac{\coth[\pi x]}{x} - \pi\,\mathrm{Re}\,\frac{\coth\left[\pi\sqrt{x^2+i(\Lambda R)^2}\right]}{\sqrt{x^2+i(\Lambda R)^2}}\ . \tag{10}$$

Again, the single term in (9) is convergent, but it is more difficult to extract the leading Λ dependence. In the limit in which $\Lambda \to \infty$ one obtains the same result as in (8).

The same computation can be performed using a suitable adapted version of dimensional regularization (see appendix D of [8]). Even if dimensional regularization is not sensitive to power-like divergences, it is useful to check that it reproduces the PV result. After extending the integral and the series to generic dimensions d and δ, introducing two Schwinger parameters t_1, t_2, the Higgs mass can be written as

$$\begin{aligned} -i\,m^2_{\phi_H} =& \frac{iN_c f_t^2}{R^2}\,\frac{d\,2^{d-5}\pi^{d/2}}{(2\pi)^d} \quad\times \\ & \int_0^\infty dt_1 \int_0^\infty dt_2\,\frac{1}{(t_1+t_2)^{1+d/2}}\left[\theta_3^\delta\left(\frac{it_1}{\pi}\right)\theta_3^\delta\left(\frac{it_2}{\pi}\right) - \theta_2^\delta\left(\frac{it_1}{\pi}\right)\theta_2^\delta\left(\frac{it_2}{\pi}\right)\right]\ ; \end{aligned} \tag{11}$$

where the theta functions $\theta_{2,3}$ are defined as

$$\theta_2(it) = \sum_{k\in\mathbb{Z}} e^{-\pi t(k-1/2)^2}\ , \qquad \theta_3(it) = \sum_{k\in\mathbb{Z}} e^{-\pi t k^2}\ .$$

From the asymptotic expansion for $\theta_{2,3}$ when $t_i \to 0,\infty$ it is clear that the integral in (11) is convergent when $d \to 4$, $\delta \to 1$; we checked numerically that for $d = 4$, $\delta = 1$ the result for $m^2_{\phi_H}$ coincides with (8).

Let us now discuss the interpretation of our result. Although the model under examination does not possess, after the orbifold projection, any global supersymmetry, it is nevertheless invariant under a local supersymmetry, which,

from the 4D point of view, is different at different points along the compactified direction. To obtain meaningful results from a loop calculation, it is therefore necessary to respect such a symmetry. The natural and simplest way to take into account the corresponding constraints is to introduce a (local) supersymmetric regulator. We stress that any sensible regularization must render finite both the sum and the integral and moreover there is no reason to distinguish between them, being on the same footing from a 5D point of a view. It is here that our interpretation diverges from ref [5]. Local supersymmetry would not be respected if the sum over the KK modes were cut at a finite value, much in the same way a sharp cut-off would break gauge invariance in QED. Hence the appearance, in such a case, of a power divergence in an operator not consistent with the symmetry itself; for instance, a local mass counterterm localized in the orbifolds fixed points does not respect the residual $N = 1$ supersymmetries S and S'. It is worth to stress that UV divergences in field theory, being local, are controlled by local symmetries rather than global ones. To further clarify this point, that has nothing to do with SUSY but is much more general, consider a scalar field theory in 5D with the fifth dimension compactified to a circle S^1. Of course the global $SO(4,1)$ invariance is broken to $SO(3,1) \times U(1)$ by the compactification, nevertheless the classical action is invariant under the diffeomorphisms of $M^4 \times S^1$ and only general covariant counterterms like $g^{MN}\partial_M\phi\partial_N\phi$ are allowed. For instance a term like $(\partial_5\phi)^2$ which is invariant under $SO(3,1)\times U(1)$ is forbidden. In the case of an orbifold compactification the situation is similar with a slight complication due to the presence of boundaries. Notice that in this discussion gravity is *not* dynamical but simply an external background. When SUSY is present, diffeomorphisms are promoted to super diffeomorphisms and the same result applies. Of course if one uses a sharp cut-off this picture is lost. Summarizing, provided that the relevant symmetries are preserved, differently from what claimed in [5], we have shown that the Higgs mass term induced by localized Yukawa couplings on the orbifold fixed points is finite and unambiguously defined.

During the completion of this work another paper appeared [9] on the same subject, reaching the same conclusions by means of a thick brane. In this respect, we notice that our method seems more general, because it is more in the spirit of effective field theory, in which brane thickness cannot be resolved, and suitable for a systematic treatment of divergences as in conventional field theory.

Note Added

After this work was completed a new paper appeared [10]. It was pointed out that in the specific model considered by Barbieri, Hall and Nomura (BHN) there is a UV sensitivity in the Higgs mass stemming from a Fayet-Iliopoulos (FI) term which arises at 1-loop. The FI term signals the presence of a gauge anomaly which could render the BHN model inconsistent. However, our explicit computation of the 1-loop Higgs mass correction involves only the

Yukawa sector and therefore it is not altered. Moreover, we stress that our discussion on the structure of counterterms which result from the choice of a regulator consistent with the underlying symmetries has a general validity, provided that anomalies are absent.

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